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Synchronization and control in a unidirectionally coupled array of chaotic diode resonators

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We experimentally investigate the synchronization of a one dimensional array of unidirectionally coupled chaotic diode resonators. Though care is taken to match diode elements as closely as possible, slight differences in the diode characteristics unavoidably affect the quality of the synchronization between the first and last element of the chain. We explore two different approaches for synchronizing each new element added to the chain and its effect on the quality of the end-to-end synchronization as measured by the mutual information between the end elements. In addition, once the chain is synchronized, we apply chaotic control by means of occasional proportional feedback to the first element. We are able to stabilize the period-1 through period-10 unstable periodic orbits in each element of the synchronized array.

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I. INTRODUCTION

Recently there has been intense interest in the study of spatiotemporal synchronization of nearly identical, coupled chaotic systems [1]. Investigations have explored synchronization in globally coupled Lorenz-like systems [2], diffusively coupled electronic circuits [3,4], and in arrays of diffusively coupled logistic maps [5]. The growing interest in synchronizing chaotic signals is driven by the potentiality for applications to secure communications [6], designing arrays of coupled chaotic lasers [7], developing cardiac pacemakers, and other biomedical applications [8].

Various coupling schemes have been employed to explore synchronization including mutual coupling, unidirectional coupling, and global coupling among the chaotic elements. The case of *unidirectional* coupling has been investigated experimentally by Newell *et al.* [9] utilizing two nearly identical driven diode resonators and by Rul'kov *et al.* [10] using two nearly identical autonomous chaotic circuits. The former authors showed that the method of continuous unidirectional feedback requires only small amounts of feedback relative to the amplitude V_0 of the driving signal. Further, once synchronization had been established, it continued to be maintained while V_0 was scanned through a large range of values encompassing a period doubling cascade to chaos.

In this paper we focus on a one dimensional array of chaotic diode resonator circuits in which each successive element in the chain is coupled to the previous element by a unidirectional continuous feedback term to be described below. We are interested in the quality of the synchronization between the first element (master) and the last slave element. One would expect that synchronization should be perfect for an array of identical elements. In practice, this is almost never realized due to inevitable and unavoidable variations of component elements from circuit to circuit. In this latter case, the quality of the synchronization between the first and last element is dependent on the method used to synchronize each additional element to the synchronized chain.

This paper is organized as follows. In Sec. II we briefly describe the synchronization and control methods utilized in our experiments. In Sec. III we describe two experiments which employ different methods to optimize the synchronization of each new element added to the chain of diode resonators. We compute the mutual information between the first and last elements as a measure of the quality of the end-to-end synchronization of the array. Once the chain has been synchronized, we experimentally show that controlling an unstable periodic orbit (UPO) in the first (master) element propagates this control down the entire chain of slave elements, so that all elements track the same controlled UPO. Finally, in Sec. IV we present our summary and conclusions.

II. SYNCHRONIZATION AND CONTROL METHODS

In this work, successive diode resonator circuits are synchronized to their immediately preceding upstream neighbor by the method of unidirectional continuous feedback [11], Fig. 1(a). In general, this method entails the coupling of two chaotic dynamical systems $\boldsymbol{x}$ and $\boldsymbol{y}$

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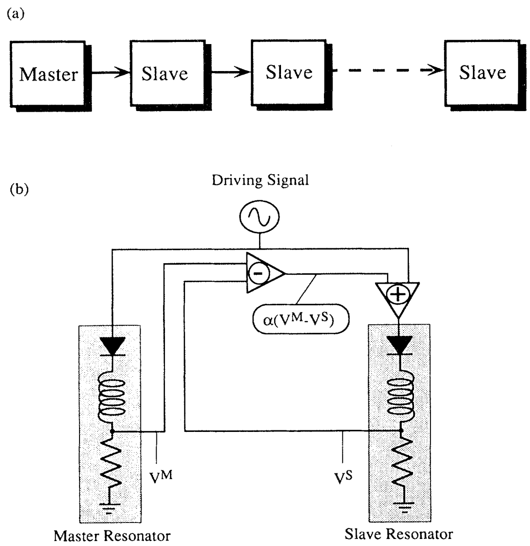

FIG. 1. A block diagram of the unidirectionally coupled array is shown in (a). The ith element is the master of the (i+1)th; the (i+1)th element exerts no influence over the ith. Our array consists of 12 resonators in total. (b) A schematic of the coupling procedure. An AD521 instrumentation amplifier generates $\alpha_i\,[V^{(i)}(t) - V^{(i+1)}(t)]$ which is summed to the driving wave of only the slave.

by a term(s) linear in the difference between a component(s) of the two signals via

$$\dot{\boldsymbol{x}} = \boldsymbol{F}(\boldsymbol{x}, t),$$
$$\dot{\boldsymbol{y}} = \boldsymbol{F}(\boldsymbol{y}, t) + \boldsymbol{K} \cdot (\boldsymbol{x} - \boldsymbol{y}). \tag{1}$$

In most instances the coupling matrix $\boldsymbol{K}$ is diagonal with one (or more) nonzero entries. Newell *et al.* experimentally demonstrated the synchronization of two chaotic diode resonators utilizing this scheme which coupled the voltages across the circuit resistors [9].

The chaotic control utilized in this work is Hunt's [12] occasional proportional feedback (OPF) variation of the control method developed by Ott, Grebogi, and Yorke (OGY) [13]. We define a Poincaré section as the maxima of the voltage across the resistor of the master resonator. This, in turn, defines a mapping (the first return map) between successive voltage maxima V_n via $V_{n+1} = \mathcal{F}(V_n, p)$ which, in general, depends on a control parameter p. The OPF perturbation δp necessary to control the unstable periodic orbit V_F is given by

$$\delta p_n = \alpha\,(V_n - V_F) \tag{2}$$

for some constant α. The algorithm of OGY gives an explicit prescription for calculating α from the rate of divergence of neighboring orbits and the sensitivity of the location of V_F to the control parameter p. Hunt's OPF variation of the OGY method lumps these structural factors into a constant α which can be found empirically. In addition, V_F is replaced with a reference voltage level V_{ref}. By varying V_{ref} and α a large number of UPOs can be stabilized. This method afforded an approach which was easily implemented in our experiments.

III. EXPERIMENT AND RESULTS

A. Setup

A block diagram of the unidirectional coupling experiment is shown in Fig. 1(b). Our chaotic circuit, the diode resonator (see Ref. [14]), is composed of a 1N4007 pn-junction diode in series with a 33 mH inductor and a 90 Ω resistor. Each resonator is driven with an amplitude of 5.4 V, in phase and sinusoidally at 70 kHz by the same wave form generator. Chaos is exhibited in the current through the circuit which we measure as the voltage drop across the resistor, $V(t)$. In a unidirectional coupling experiment, feedback consists of a time varying, proportional amount $\alpha_i\,[V^{(i)}(t) - V^{(i+1)}(t)]$ where the superscripts i and $i+1$ refer to the ith and (i+1)th resonators, respectively, and α_i refers to the amplification factor for the ith pair. This feedback is summed to the drive wave of the (i+1)th resonator only, i.e.,

$$V^{(i+1)}_{drive}(t) = V_0\,\sin(\omega t) + \alpha_i\,[V^{(i)}(t) - V^{(i+1)}(t)]. \tag{3}$$

The feedback term for each couple is generated using an AD521 instrumentation amplifier. The quantity α_i is determined experimentally by adjusting the gain of the appropriate instrumentation amplifier until the ith and (i+1)th chaotic signals lock together. We drove the first resonator, designated as the master, with a WaveTek 166 wave form generator. The ten other resonators, unidirectionally coupled so that the ith element is master to the (i+1)th, are all driven with a HP3325A wave form generator which is triggered in phase with the WaveTek wave form generator. To prevent undesired crosstalk through the drive waves, each resonator was buffered from the others with an LH0002 low impedance line driver.

In order to identify identical resonators, we temperature stabilized some 200 diodes and then observed their various properties. From these, we selected 12 diodes whose characteristics were closest. We then mated these chosen few with matching inductors and resistors. Even then, the orbits of the attractor varied between each resonator not only from the outset, but also from a day to day basis. Differences in the attractors can be seen in Fig. 2(a) which shows four first return maps superimposed. Since the attractors are only partially identical, the trajectories in a given pair of resonators will not be exactly the same. Therefore we do not expect exact synchronization between a neighboring pair of resonators, let alone complete synchronization across the whole array.

In the OPF control method we used to stabilize the periodic orbits, the empirically obtained α amplifies $V_n^{(1)} - V_{ref}$, Eq. (2), where $V_n^{(1)}$ refers to the nth voltage peak of the first (master) circuit and V_{ref} is a reference voltage. When $|V_n^{(1)} - V_{ref}|/V_0$ is small, the factor α

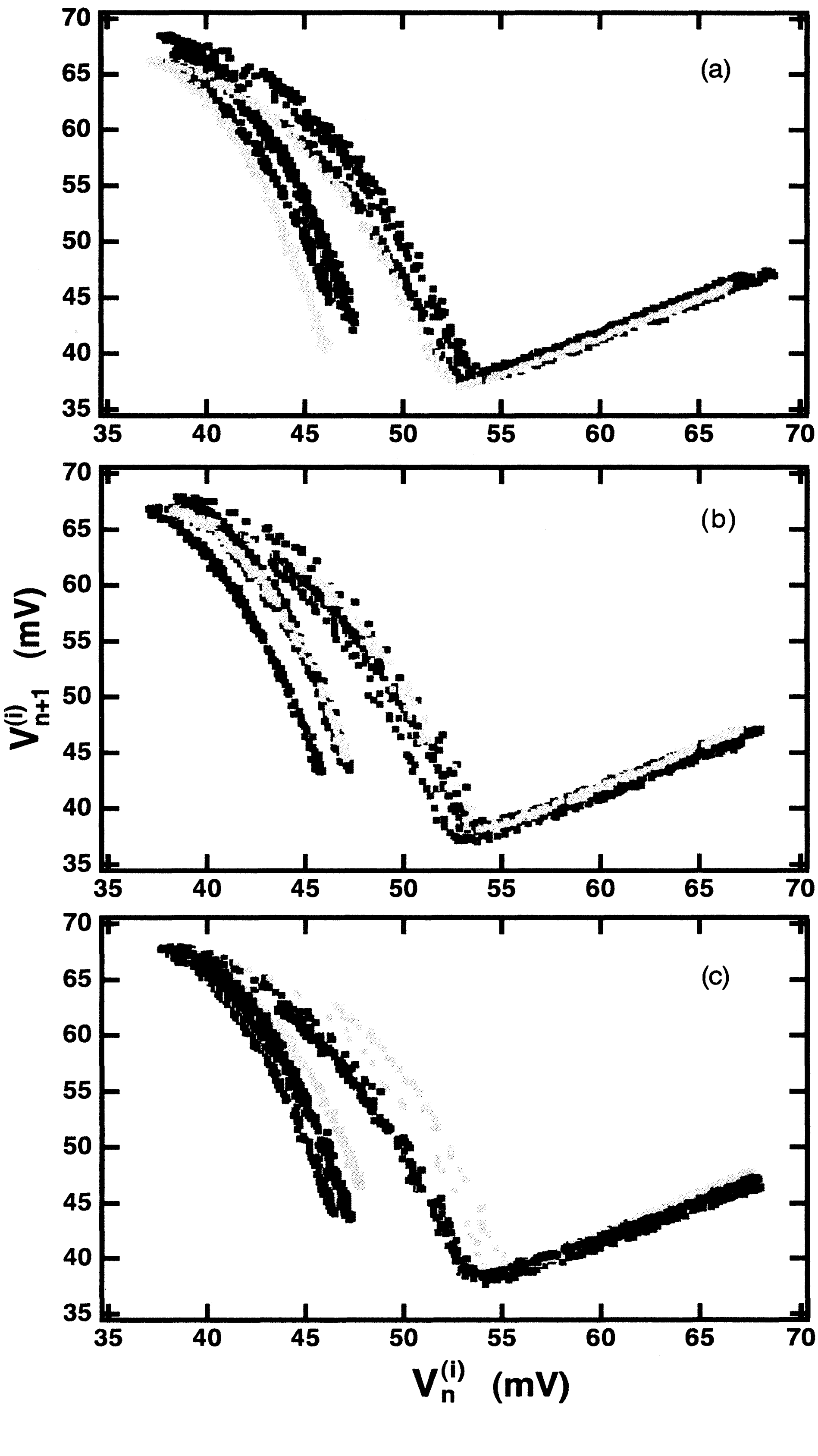

FIG. 2. First return maps are plotted for an array of four elements. (a) With no coupling between each element, though each return map is similar in form, orbits which exist for one attractor do not exist for others. (b) The elements are coupled so that synchronization is optimized between the ith and $(i+1)$th element (red → blue → yellow → green). (c) When coupled so that the fourth element (green) is optimally synchronized to the first (red), the attractors of the second (blue) and third element (yellow) are noticeably perturbed. The attractor of the fourth element (green) now more closely coincides with that of the first (red).

modulates the driving sine wave for a fraction of the period, otherwise no feedback signal is applied.

B. Experiments

In the first experiment, we optimally synchronized the first slave to the master, the second slave to the first, the third slave to the second, and so on down the chain. In this *pairwise* coupling process, we were only interested in the optimization of the synchronization between nearest neighboring elements. That is, when an $(N+1)$th element was added to the chain of N synchronized elements, it was optimally synchronized to the last, Nth element. As each new element was added to the chain and synchronized to the last element, we computed the mutual information between the master and last slave resonator from experimentally captured time series. The results are shown in Fig. 3. This graph plots the mutual information [15] (the ordinate) between the master resonator and each slave (indicated along the abscissa). There is considerable walkoff as the number of elements in the array increases. This is due to the fact that since we are attempting to synchronize the orbits only between adjacent resonators, i.e., locally, we are ignoring global relationships among the attractors. Thus the slight physical differences in the resonators cause the quality of the end-to-end synchronization to degrade as the number of elements in the chain increases. Conclusions from this first test indicate that extremely exacting standards should be implemented when designing arrays.

The second experiment attempts to rectify problems of the first. As each new element was added to the end of the chain we optimized its synchronization to that of the master. This was obtained by varying the α gain factors between each pair of resonators in the chain until the synchronization between the first and last element was optimized. As each new element was added to the end of the chain, this procedure of altering intermediate gains to optimize the end-to-end synchronization was repeated. As seen in Fig. 4, the result yielded a clear amelioration of the global, end-to-end synchronization. The net effect of altering the gain between intermediate elements in the chain was to shift the attractor of the last slave element closer to that of the master element. The improved synchronization between the end elements was achieved at the expense of a decrease in the quality of the synchronization between some intermediate neighboring elements. A comparison of Fig. 4 with Fig. 3 shows that the end-to-end synchronization optimization procedure for each new added element resulted in an overall improvement in the net mutual information for the entire array.

In a subsequent experiment we explicitly demonstrate the observed shifting of the attractors in a synchronization experiment utilizing a four element array. Figure 2(a) shows the four return maps superimposed while uncoupled. We then applied the pairwise coupling scheme. Figure 2(b) shows the effect on the return maps. We point out that though our feedback represents less than 3% of the drive wave, there is a noticeable shifting of the orbits. The return map of the last element is relatively different from that of the first and synchronization

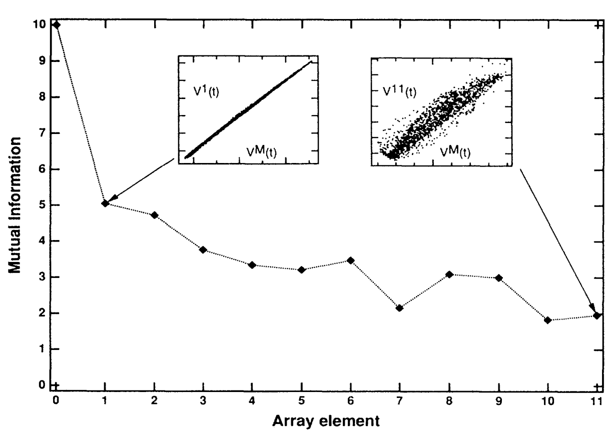

FIG. 3. Mutual information between first and last elements of a chain of chaotically synchronized diode resonators as a function of the number N of slave elements in the chain. Synchronization was optimized in a pairwise manner. That is, as each new element N was added to the existing chain of $N-1$ slave elements, the synchronization was optimized between the $(N-1)$th and Nth element. The insets show the synchronization between first and last element for a chain of $N=1$ and $N=11$ slaves. Note that in the latter case, $N=11$, the synchronization between the first and last element is severely degraded over the former case, $N=1$.

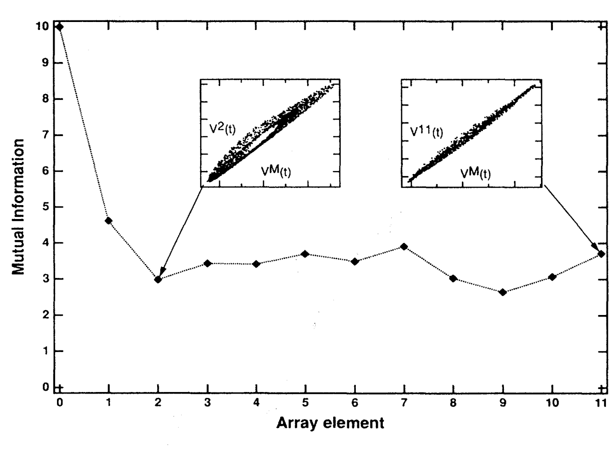

FIG. 4. Mutual information between the master and each slave element in a chain of chaotically synchronized diode resonators with 11 slave elements. Synchronization was optimized in a *global* manner, i.e., the synchronization was optimized between the first and 11th element by altering the gains between intermediate pairs of elements. The insets show the synchronization between the master and the second slave (left inset) and between the master and the 11th slave (right inset). In order to optimize the synchronization between the master and the 11th element, the synchronization between the master and the second element was degraded.

is not especially well obtained. We next optimize the synchronization between the first and last element, Fig. 2(c). In doing so, the return map of the last element is pulled towards the first and synchronization is improved. However, in doing so, the return map of the third element is substantially altered; synchronization between the third and fourth elements is weakened.

In a final experiment, we have also demonstrated that the chaos in the synchronous array could be controlled by OPF. Once the entire array was synchronized (with the latter optimization scheme described above), controlling pulses of the form given by Eq. (2) were applied to amplitude modulate the drive wave of the master element alone. Since the master was driven by a separate wave form generator than the slave resonators, no controlling pulses were directly applied to the slaves. When the master resonator was stabilized, the slaves synchronized onto the controlled periodic orbit. In this manner we successfully controlled and synchronized the first through the tenth periodic orbits. Figure 5 shows the results obtained for the period-3 orbit and the period-10 orbit. We first extracted the period-3 orbit from a time series of the chaotic resonator by the method of close returns [16]. Figure 5(a) shows the extracted time series while Fig. 5(b) portrays this series in time delay coordinates. Controlling pulses were then applied to the chaotic master. The controlled period-3 orbit for the master is shown in Fig. 5(c) (as a time delayed plot) while the synchronized and controlled last slave is shown in Fig. 5(d). We see that these are hardly perturbed from the extracted orbit. The extracted period-10 orbit is shown as a time series in Fig. 5(e) and in delay coordinates in Fig. 5(f). Figures 5(g) and 5(h) show the time delayed orbits of the master and last slave, respectively. As a final note, some unstable periodic orbits of period greater than 10 were also extracted and stabilized. However, no systematic effort was made to control the orbits of period greater than 10.

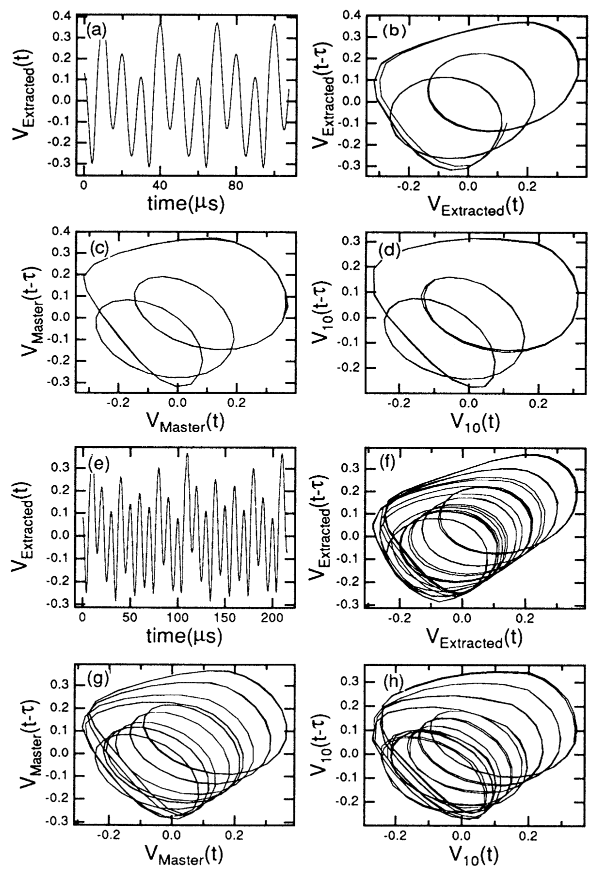

FIG. 5. The stabilized period-3 and period-10 orbits are displayed. (a) The extracted period-3 orbit from a time series of the chaotic resonator. (b) The extracted period-3 orbit in time delay coordinates. (c) The controlled period-3 orbit for the master. (d) The synchronized and controlled period-3 orbit in the last (11th) slave. The extracted period-10 orbit from the chaotic resonator is shown in (e) and in time delay coordinates in (f). (g) and (h) show the time delayed orbits of the controlled master and last slave, respectively.

IV. SUMMARY AND CONCLUSIONS

In principle, a large linear array of identical chaotic circuit elements should achieve perfect synchronization through the method of continuous feedback. However, even with careful selection of the component elements, perfect synchronization is almost never realized in practice. We have shown here that the seemingly reasonable straightforward approach of optimizing the synchronization between a newly added element and the last element in the linear array leads to poor synchronization between the first and last element as the size of the array increases. The unavoidable component variations in the circuit elements lead to slightly different attractors, which by the time we reach the last element can differ significantly from that of the first element. By optimizing the synchronization between the first element and the newly added element by adjusting the gains between intermediate elements in the chain, the end-to-end synchronization was improved. This approach also led to a better overall synchronization for the entire array itself.

Once the array was synchronized we were able to stabilize the UPOs of up to period 10 throughout the entire array by applying chaotic control to the first element. The marriage of chaotic control and synchronization techniques could have important applications to creating coherent arrays of coupled lasers. This work points out that unavoidable variations in "off-the-shelf" components can limit the ultimate size of a synchronized array of such elements if they are simply added in line and synchronized to the last element of the array. More care needs to be taken if a good end-to-end and overall synchronization is to be achieved. We have presented one such method of achieving a better overall synchronization by optimizing the newly added element to the first element with the subsequent alteration of the gains of the intermediate elements.

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[1] A partial listing of the vast number of papers in this field can be found in the paper by J.F. Heagy, T.L. Carroll, and L.M. Pecora, Phys. Rev. E **50**, 1874 (1994) and in the review article by H.D.I. Abarbanel, R. Brown, J.J. Sidorowich, and L.S. Tsimring, Rev. Mod. Phys. **65**, 1331 (1993); see also D. Auerbach, Phys. Rev. Lett. **72**, 1184 (1994); T.L. Caroll and L.M. Pecora, Physica D **67**, 126 (1993).

[2] J.M. Kowalski, G.A. Albert, and G.W. Gross, Phys. Rev. A **42**, 6260 (1990).

[3] See first reference in [1].

[4] P.S. Linsay and A. Brailove, in *Proceedings of the First Experimental Chaos Conference*, edited by S. Vohra, M. Spano, M. Shlesinger, L. Pecora, and W. Ditto (World Scientific, Singapore, 1992), p. 315.

[5] Y.C. Lai and C. Grebogi, Phys. Rev. E **50**, 1894 (1994).

[6] S. Hayes, C. Grebogi, and E. Ott, Phys. Rev. Lett. **70**, 3031 (1993); K.M. Cuomo and A.V. Oppenheim, *ibid.* **71**, 65 (1993); H.D.I. Abarbanel and P.S. Linsay, IEEE Trans. Circuits Syst. II **40**, 643 (1993); S. Hayes, C. Grebogi, E. Ott, and A. Mark, Phys. Rev. Lett. **73**, 1781 (1994); P. Colet and R. Roy, Opt. Lett. **19**, 2056 (1994).

[7] H.G. Winful and L. Rahman, Phys. Rev. Lett. **65**, 1575 (1990); R. Roy and K.S. Thornburg, Jr., *ibid.* **72**, 2009 (1994); T. Sugawara, M. Tachikawa, T. Tsukamoto, and T. Shimizu, *ibid.* **72**, 3502 (1994).

[8] A. Garfinkel, M.L. Spano, W.L. Ditto, and J. Weiss, Science **257**, 1230 (1992); S.J. Schiff, K.J. Jerger, D.H. Duong, T. Chang, M.L. Spano, and W.L. Ditto, Nature (London) **370**, 615 (1994).

[9] T.C. Newell, P.M. Alsing, A. Gavrielides, and V. Kovanis, Phys. Rev. E **49**, 313 (1994).

[10] N.F. Rul'kov, A.R. Volkovskii, A. Rodríguez-Lozano, E. Del Río, and M.G. Velarde, Chaos Solitons Fractals **4**, 201 (1994).

[11] K. Pyragas, Phys. Lett. A **181**, 203 (1993).

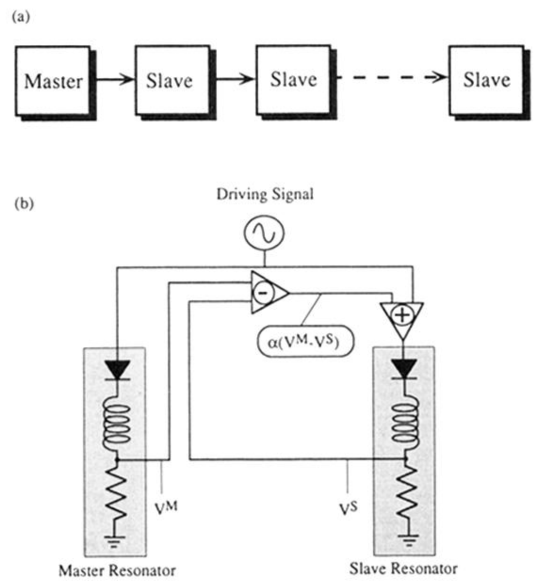
[12] E.R. Hunt, Phys. Rev. Lett. **67**, 1953 (1991).

[13] E. Ott, C. Grebogi, and J.A. Yorke, Phys. Rev. Lett. **64**, 1196 (1990).

[14] For a review of the diode resonator and pertinent articles, see T.C. Newell, P.M. Alsing, A. Gavrielides, and V. Kovanis, Phys. Rev. E **51**, 2963 (1995), and references therein.

[15] The idea of the mutual information is drawn from the information theory work of C.E. Shannon, Bell Syst. Tech. J. **27**, 379 (1948). In short it is an indication of the information that one knows about point $x(t+T)$ given point $x(t)$. When applied to synchronization, the mutal information, also known as the redundancy, is a measure of the information known about $y(t)$ when given $x(t)$.

[16] G.B. Mindlin, H.G. Solari, M.A. Natiello, R. Gilmore, and X.J. Hou, J. Nonlinear Sci. **1**, 147 (1991); D.P. Lathrop and E.J. Kostelich, Phys. Rev. A **40**, 4028 (1989).



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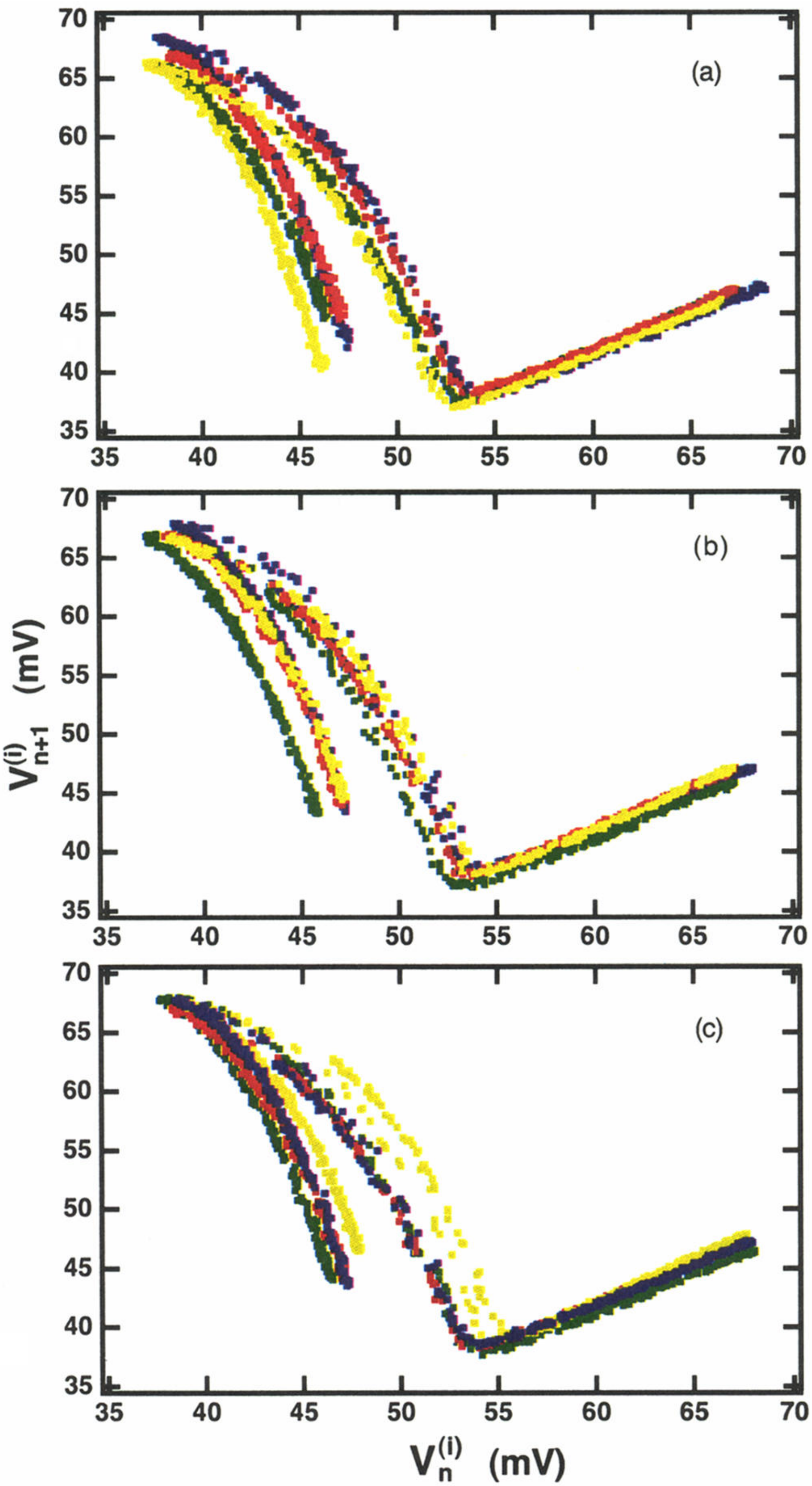

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